Pricing for Heterogeneous Services in OFDMA 802.16 Systems

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***Abstract*—The IEEE 802.16 standard is expected to become a successful broadband wireless access solution, especially thanks to its ablility to provide high data rates, to differentiate services, and thanks to its interoperability, as promoted by the Worldwide Interoperability for Microwave Access (WiMAX) organization. We consider in this paper an Orthogonal Frequency Division Multiple Access (OFDMA)-based 802.16 operator, and investigate how to charge in order to control demand and maximize the revenue. Pricing has indeed been seen as a way to provide return on investment for providers, as well as to control demand and differentiate services for delivering a satisfactory quality of service (QoS) to heterogeneous applications. Our model specifically assumes two classes of applications with an infinite population of potential customers. The average number of customers per class is naturally regulated by prices but also the resulting QoS, this QoS depending itself on the customers in each class. We analyze the equilibrium situation of this game for fixed prices, and then numerically determine prices maximizing the operator's revenue.**

I. Introduction

Internet wireless communications have experienced a tremendous growth during the last years thanks to new standards such as IEEE 802.11 (WiFi). But at the same time, there has been a growth in terms of services, including Voice over IP, peer-to-peer applications... WiMAX (Worldwide Interoperability for Microwave Access) is an IEEE 802.16 standardized technology (see [1]) providing high data rates over long distance (up to 30 miles), and therefore covering areas where it is too costly for operators to develop wired infrastructures. This economic interest is enhanced by the ability to support QoS (Quality of Service) for different types of service, as well as its interoperability. WiMAX indeed supports all categories of services under real and non-real time communications, real time flows being served by unisolicited grant service (UGS or real-time-polling services (rtPS), while non-real time services are divided in two categories of traffic, non-real-time-polling services (nrtPS) and best effort services (BE). We consider in this paper the OFDMA (Orthogonal Frequency Division Multiple Access) 802.16 [2], which is preferable for non-line-of-sight applications.

While performance evaluation of WiMAX, and its admission control, have received a lot of attention in recent years (see among others [3], [4], [5]), there is surprisingly, to our knowledge, no careful study about how pricing could be used to control demand and efficiently differentiate services. Indeed, what would be the point of providing different class of services if there is no way to limit the access to the classes offering th best QoS? Pricing is a way to perform that control and to help the operator to maximize its revenue. It has been studied in wired telecommunications (see [6], [7] for surveys), in CDMA-based wireless communications [8], [9], as well as for WiFi access [10].

Our goal, and contribution, is to introduce a pricing scheme for heterogeneous services for an OFDMA 802.16 communications operator. We consider two classes of applications with different, but elastic, QoS requirements. These requirements are represented by the so-called *utility functions*. We consider that demand exceeds capacity in general so that demand will be regulated by prices and available QoS. In order to model the QoS in OFDMA 802.16 systems, we use the performance evaluation model in [4] where interferences and loss probabilities are computed analytically for *given traffic loads*. The load will here be driven by the above utility functions: customers apply for service as soon as they get a positive utility. Load therefore depends available QoS, itself dependent of the load. This results in an equation to be solved. Moreover, QoS for a given class depends on the number of customers of the other class. This introduces a game, and the existence (and uniqueness) of an equilibrium point requires a careful analysis. This analysis being performed for fixed prices, we then in a second step investigate the prices which maximize the operator's revenue, taking into account the corresponding equilibrium loads.

The paper is organized as follows. Section II describes the performance and utility models. Section III analyzes, for fixed prices, the equilibrium in the game on the number of customers in each class. Knowing this equilibrium, Section IV the looks at the prices which yields a maximal revenue for the operator. Finally, Section V presents our directions for future research.

II. The model

This section presents the basic model we will use throughout the paper. More precisely, Section II-A summarizes the performance evaluation results of [4] to describe OFDMA 802.16 systems, with additional definitions of performance measures that we will use, while Section II-B presents the general utility model that will give the number of customers for each class.

A. Performance model for OFDMA 802.16

Suppose that we have L classes of calls (we will later restrict to $L = 2$ classes). Consider a multi-cell network, with n cells, to deal with the collisions due to interfering cells, the frequency band in each cell being decomposed in N subcarriers. We review here the model for getting the proportion of class-k (for $1 \leq k \leq L$) symbols with degraded Signal to Noise ratio (SNR), $R(k)$, that is useful to get the throughput *for a given number of customers*.

Class-k calls arrive to a given cell i ($0 \leq i \leq n-1$) with Poissonian rate $\lambda_{k,i}$, and require to be assigned l_k sub-channels for an arbitrary distributed service time with mean $1/\mu_{k,i}$. Note that call arrival rate $\lambda_{k,i}$ will be variable and correspond to the demand for class-k calls. Vectors

$$\mathbf{U}_i = (U_{0,i}, \cdots, U_{L-1,i}),$$

$$\mathbf{K} = (K_0 = \sum_{k=0}^{L-1} l_k U_{k,0}, \cdots, K_{n-1} = \sum_{k=0}^{L-1} l_k U_{k,n-1})$$

represent respectively the number of calls belonging to the L classes in cell i and the number of occupied subchannels in the n cells. Then the probability of having $U_{k,i}$ class-k calls in cell i, $\pi(\mathbf{U}_i)$, is given by:

$$\pi(\mathbf{U}_i) = \frac{1}{G} \prod_{k=0}^{L-1} \frac{(\lambda_{k,i}/\mu_{k,i})^{U_{k,i}}}{U_{k,i}!},$$

G being the normalizing constant. Similarly, $\Pi_i(K_i)$, the probability of having K_i occupied sub-channels in cell i, is

$$\Pi_i(K_i) = \sum_{\mathbf{U}_i : \sum_{k=0}^{L-1} l_k U_{k,i} = K_i} \pi(\mathbf{U}_i).$$

We also assume that the path loss q_i between the cell i base station and the receiver is $q_i = r_i^{\mu} 10^{\xi_i/10}$ with r_i the distance from the base station, ξ_i a log normal random variable (shadowing) and $\mu = 4$, and that there is a degradation as soon as the SNR goes below a threshold T. The SNR requirement for a Bit Error Rate (BER) less than 10^{-6} depends on the modulation type and is specified within the standard [1].

Under those assumptions, it is shown in [4] that:

- The probability of SNR degradation when collision occurs between subcarriers of cells 0 and i is

$$p_{0i} = E_X[\frac{1}{2} erfc(\frac{-5\ln(TX^{\mu})}{\varsigma b \ln(10)})]$$

  where $erfc(\cdot)$ is the complementary error function of a standard normal distribution and X is the random variable representing the ratio, in each position of cell 0, of the distances to base stations 0 and i ($X = \frac{r_0}{r_i}$). The expectation is taken over the surface of cell 0. In our calculations, we consider standard path loss parameters ($b = 1/\sqrt{2}$, ς = 8 dB and μ = 4).
- When the frequency band of N subcarriers is partitioned into M clusters, the expected number of collisions given the vector $\mathbf{K}$ of occupied subcarriers, $E_{\mathbf{K}}[c]$, is $E_{\mathbf{K}}[c] = M \cdot E'_{\mathbf{K}}[c]$, with $E'_{\mathbf{K}}[c]$ the expected number of collisions in one cluster given by $E'_{\mathbf{K}}[c] = \sum_{c \in C} c P'_{\mathbf{K}}(c)$, $P'_{\mathbf{K}}(c)$ being the probability of having c collisions, defined for $\max(0, K_0 + \max_{i \neq 0}(K_i) - \frac{N}{M}) \leq c \leq \min(K_0, \min(\frac{N}{M}, \sum_{i=1}^{n-1} K_i))$,

$$P'_K(c) = \binom{K_0}{c} \left(1 - \prod_{i=1}^{n-1} \frac{N/M - K_0 - K_i + c}{N/M - K_0 + c}\right)^c \prod_{i=1}^{n-1} \left(\prod_{j=0}^{K_i - 1} \frac{N/M - (K_0 - c) - j}{N/M - j} \right).$$

- Finally, the proportion of symbols with degraded SNR for class-k calls is

$$R(k) = \sum_{\mathbf{U_0}} \frac{U_{k,0} l_k}{K_0} \pi_0(\mathbf{U_0}) [\sum_{\bar{\mathbf{K}}} \left(\prod_{i=1}^{n-1} \Pi_i(K_i) \right) R(\mathbf{K})]$$

  where $\bar{\mathbf{K}} = (K_1, \ldots, K_{n-1})$ and $R(\mathbf{K})$ is the symbol error rate given $\mathbf{K} = (K_0, K_1, \ldots, K_{n-1})$ equal to

$$R(\mathbf{K}) = \frac{\sum_{i=1}^{n-1} p_{0,i} K_i}{\sum_{i=1}^{n-1} K_i} \frac{E_{\mathbf{K}}[c]}{K_0 M}.$$

An additional performance measure that we introduce here and required to define the thorughtput with respect to what users aim at sending is the blocking probability $B_{k,i}$ of a given class of service-k in cell i:

$$B_{k,i} = \sum_{\hat{\mathbf{U}}_{k,i}} \pi_i \left(\hat{\mathbf{U}}_{k,i} \right)$$

where a blocking state $\hat{\mathbf{U}}_{k,i}$ is defined as a state when any additional class-k call will be blocked due to lack of resources.

As a consequence, the mean throughput y_k for a class-k call will be proportional to $(\delta l_k (1 - R(k))(1 - B_k)$ where δ is the maximum throughput per sub-channel. Indeed, the obtained rate is the one per subchannel times the number l_k of used sub-channels, but this has to be multiplied by $(1 - B_k)$ the probability of not being blocked. Throughout the rest of the paper we will forget the subscript i later on since we will only focus on an arbitrary cell.

B. General utility model

Demand in each class k in a given cell i is driven by the arrival rate $\lambda_{k,i}$ and depends on the delivered QoS as we will see. We assume that potential demand always exceeds capacity. In our model, customers are supposed to be sensitive to their mean throughput. This relation is expressed by the *valuation function* $v_k(y)$ a class-k customer ($1 \leq k \leq L$) gets (expressed in financial units here) for the corresponding mean throughput y. But the larger the number of users in the network, the higher the probability of QoS degradation, because collisions can occur between subcarriers of different cells [4]. Thus, clearly, the mean throughput is affected by the number of customers asking for service in each class. This relation will be investigated.

More formally, $v_k(\cdot)$ is assumed to be an increasing function of $y_k = \delta l_k((1 - R(k))(1 - B_k)$. As in [11], we consider the valuation functions $v_k(y) = y^{\alpha_k}$. For example, assuming that service is differentiated by two classes of traffic, we have two valuation functions $v_0(y) = y^{\alpha_0}$ and $v_1(y) = y^{\alpha_1}$ with $0 < \alpha_0 < \alpha_1$. This somewhat arbitrary choice of valuations is based on the idea that valuation curves do intersect: each valuation function has equal value for $y = 1$. These valuation functions imply that the high throughput is more valuable to the higher class (class-1) than the lower class (class-0). On the other hand, the lower class is relatively insensitive to the low throughput compared with the higher class.

The *utility function* for any class-k user represents the (financial) net benefit and is therefore the difference between valuation and access price P_k,

$$u_k(\lambda_0, \ldots, \lambda_L) = v_k(\delta l_k((1 - R(k))(1 - B_k)) - P_k.$$

An important remark is that this utility depends not only on the number of users in the given class-k, but also on the number in the other classes (due to collisions), and we therefore typically are in the context of non-cooperative game theory [12].

III. Equilibrium analysis of the game

How will the demand be in each class regulated? This is typically a game between the classes of applications. Indeed, given that potential demand always exceeds capacity, the arrival rate λ_k for class-k will naturally increase (by new customers arriving) if the cost of sending calls is less than the valuation that they get from it. On the other hand, it will naturally decrease if the cost of sending calls is larger than the valuation that they get from it, because customers do not have any interest in using the resource. As a consequence, for fixed value of the other rates, the equilibrium value for rate λ_k is such that we have one of the two cases:

- Either λ_k is such that $v_k(\lambda_k) = P_k$ (which can be expressed as $u_k(\ldots, \lambda_k, \ldots) = 0$), meaning that the rate adapts itself up to saturation;
- Or $\lambda_k = 0$ because $v_k(0) < P_k$ (i.e., $u_k(\ldots, 0, \ldots) < 0$), meaning that the access price is too expensive for class-k to use the network in the current conditions.

This principle is described for a single class, but is true for the whole set of classes altogether. The existence (and uniqueness) of an equilibrium point is therefore an issue, hence the non-cooperative game-theoretic framework.

An equilibrium can be determined graphically for $L = 2$ classes. Consider the 2 dimensional domain with λ_0 and λ_1 on the two axes. Both functions $u_k(\cdot, \cdot)$ are strictly decreasing in any of their argument when the other argument is fixed, and we can draw the curves $u_k(\lambda_0, \lambda_1) = 0$ for $0 \leq k \leq 1$.

There are then three possible situations, that we illustrate for a given set of parameters in our WiMAX context, in the case of the previously described example functions.

- Either the curve $u_0(\lambda_0, \lambda_1) = 0$ is always above the curve $u_1(\lambda_0, \lambda_1) = 0$, as illustrated Figure 1. In that case, demand naturally grows up to reaching (somewhere) the first curve, $u_1(\lambda_0, \lambda_1) = 0$ here. Then more class-0 cutomers enter because their utility is still positive, while, in such a case, less class-1 customers are expected because their utility would become negative otherwise. We then "slide" on the curve $u_1(\lambda_0, \lambda_1) = 0$ with increasing rate λ_0, up to the moment where $\lambda_1 = 0$. Then rate λ_0 still increases up to the point such that $u_0(\lambda_0^*, 0) = 0$. This gives the equilibrium point $(\lambda_0^*, \lambda_1^* = 0)$.
- Or the curve $u_0(\lambda_0, \lambda_1) = 0$ is always above the curve $u_1(\lambda_0, \lambda_1) = 0$, as illustrated Figure 2. We can proceed similarly to get the equilibrium point $(\lambda_0^* = 0, \lambda_1^*)$ with λ_1^* the solution of $u_1(0, \lambda_1^*) = 0$.
- The last possibility is when the curves intersect, as illustrated Figure 3. In that case the rates still increase up to reaching the first curve, $u_0() = 0$ or $u_1() = 0$; say k is the corresponding class. The rate for the other class still increases, while λ_k therefore decreases due to a negative utility, up to reaching the intersecion point. In that case, no class has any interest in changing his rate and the intersection point is the equilibrium.

If they intersect more than once, several equilibria become possible. On the other hand, in that case, some are not *stable* in the sense that if they intersect such

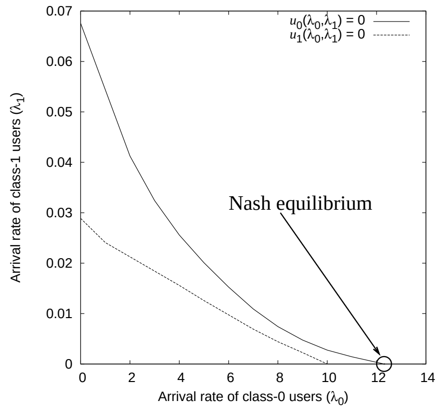

Fig. 1. Best response curves when P_0=0.55, P_1=1.75 (a_0=1.2, a_1=1.5)

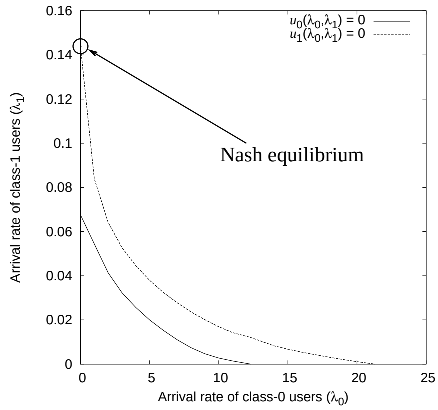

Fig. 2. Best response curves when P_0=0.55, P_1=1.35 (a_0=1.2, a_1=1.5)

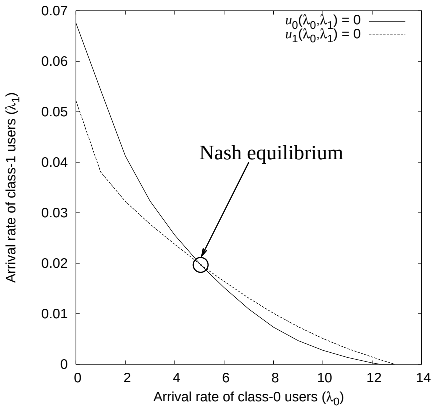

Fig. 3. Best response curves when P_0=0.55, P_1=1.15 (a_0=1.2, a_1=1.5)

that $u_0() = 0$ is first under $u_1() = 0$ and becomes the other way around, the arrival rate λ_0 still has interest in increasing because its utility then becomes positive.

To determine if there is an intersection point, one can try to solve the system (as done in [13] for another pricing context)

$$u_0(\lambda_0, \lambda_1) = v_0 \left(\delta l_0 \left(1 - R(k)\right)\left(1 - B_k\right)\right) - P_0$$
$$u_1(\lambda_0, \lambda_1) = v_1 \left(\delta l_0 \left(1 - R(k)\right)\left(1 - B_k\right)\right) - P_1$$

and look if there is a solution with $\lambda_0 \geq 0$ and $\lambda_1 \geq 0$. If it is not the case we are in one of the two first situations.

IV. Price Optimization

From the network provider's perspective, the main concern is to determine P_0^*, P_1^* maximizing the network revenue $R(P_0, P_1) = \lambda_0^* P_0 + \lambda_1^* P_1$ with $P_0, P_1 \geq 0$.

As it seems already intractable to determine analytically the equilibrium demand (λ_0^*,λ_1^*), getting the optimal prices is intractable as well. But the optimal prices can be obtained numerically. Figure 4 displays the revenue in terms of the prices when a_0=1.2, a_1=1.5. For this example, we obtain the optimal price set $(P_0^*, P_1^*) = \{(0.65, 0.95), (0.75, 0.95), (0.85, 0.95)\}$, giving an optimal revenue $R^* = R(P_0^*, P_1^*) = 7.24945$.

On the optimal prices, the curve $u_1(\lambda_0, \lambda_1) = 0$ is always above the curve $u_0(\lambda_0, \lambda_1) = 0$, similarly with Fig. 2. In such conditions, a positive valuation is provided for only class-1 customers; thus only class-1 customers exist in the system. The system parameters for numerical analysis presented in this paper are $l_0 = 1$, $l_1 = 2$, $\alpha_0 = 1.2$, $\alpha_1 = 1.5$, $\mu_k = 1/120$ and a Required SNR of 18 dB (for 64-QAM 1/2).

V. Conclusions

In this paper, we have designed a pricing scheme for OFDMA 802.16 systems, taking into account their performance characteristics, for heterogeneous applications.

Future work could head into several directions. First, we could push forward the case of more than $L = 2$ classes. Also, the paper has implicitely assumed that service is made of two classes, and that each type of application is assigned to a specific class. The case of *open classes* where the users can also use the other classes could be investigated.

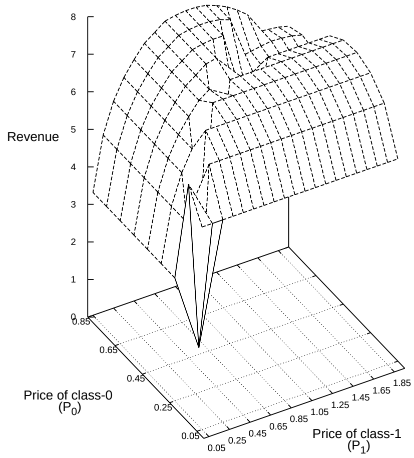

Fig. 4. Optimal provider's revenue in terms of prices when a_0=1.2, a_1=1.5

Acknowledgment

This work has been supported by ANR WiNEN project.

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